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A Study on Prediction of Economic Data using Chaos Analysis

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Abstract: The movement of financial time series such as the stock prices and currency exchange was analyzed by applying linear theory to the economic data. The portion of the change which deviated from that predicted by linear theory was handled as an error term. However analysis using only linear theory is inadequate and caused by complex parameters. Recently, deterministic chaos theory has been applied to irregularly changing time series data in fields such as physics, biology, engineering and social science. Examples include weather, earthquakes, sunspots, and so on. It has been shown that such phenomena can be more accurately analyzed and forecasted by using the chaos theory. In this study we examines whether chaotic behavior exists in financial time series data such as stock prices by using Lyapunov exponents and Lyapunov dimensions, and attempt to predict the financial index from chaos theory.

Keywords: Chaos, Lyapnov, Prediction, Financial Index.

I. INTRODUCTION

Deterministic chaos theory has been applied to irregularly changing time series data in fields such as physics, biology, engineering and social science. It is clarified that phenomena of weather, earthquakes, sunspots, and so on can be more accurately analyzed and forecasted by using the chaos theory.

The linear theory has been adopted to analyze the movement of financial time series such as the stock prices and currency exchange. However analysis using only linear theory is inadequate, because the financial time series are influenced by complex parameters and the portion of the change which deviated from that predicted by linear theory was handled as an error term.

In this study we examines whether chaotic behavior exists in financial time series data such as stock prices by using Lyapunov exponents and Lyapunov dimensions, and attempt to predict the financial index from chaos theory.

II. Characteristics of Chaos Theory [1][2][3]

Chaos state ordinarily means the confusion condition that can be dealt with, but the chaos theory analyzes the phenomenon that irregularly behaves according to time and also is controlled by simple rules.

There are two characteristics of chaos theory. The one is un-stability of trajectory that the difference of initial conditions influences the final result according to time, and in addition, the difference of the results becomes large according to the exponential function. The initial rate of change is quantified by Lyapnov index and the long term prediction is unable by chaos theory, because the difference between real value and estimated value becomes large by un-stability of trajectory. But the short term estimation can be done by the proper model.

The other characteristic of chaos theory is the boundedness that duplicates irregular movement in a short range. Chaotic behavior exits in the bounded region by the recursive motion of the nonlinear return. From this characteristic, chaotic attractor is generated. The error of the initial change is enlarged and emanated only by the un-stability of trajectory. Therefore, chaotic behavior is caused by these characteristics, the un-stability of trajectory and the boundedness of fluctuation.

The logistic map is the representative chaos. American mathematical biologist, Robert May created the logistic map in his research of the change of individual organism. The logistic map is expressed by Equation(1).

$$X(n+1) = aX(n)(1 - X(n)) \qquad (1)$$

III. Prediction by Chaos

Characteristics of chaos include the aperiodic, long-term behavior of a bounded, deterministic system that exhibits sensitive dependence on initial conditions. From this characteristic, long term prediction of time series data is not enabled by chaos.

The difference between probabilistic time series analysis and chaotic time series analysis is that the former is based on the analysis of the statistics value such as mean or variance of time series data, and on the other hand, the latter analyzes the chaotic feature quantity of Lyapunov exponents and fractal dimension.

Probabilistic time series analysis pays little attention to the structure of the subject system and by this reason the degree of accuracy of chaotic analysis is superior to statistical analysis.

The purpose of usage of chaos theory is to find the deterministic regularity from time series data and also to estimate the near future data that influenced by the data at some point.

1. Takens' embedding theorem [3][4][5]

Takens proved that the time-lagged variables constitute an adequate embedding provided the measured variable is smooth and couples to all the other variables, and the number of time lags is at least $2D+1$, where D denotes the original dimension of dynamical system.

From the Takens' embedding theorem, if k dimensional time delay vector $v(t)$ is $k > 2m$ which consists of a time series data of $y(t)$ of m dimensional dynamic system, a attractor which is reconstructed by the time delay vector is embedded in the k dimensional space. The time delay vector can be expressed by Equation(2), where τ is the time interval of sampling from time series data.

$$v(t) = (y(t), y(t+\tau), \cdots, y(t+(m-1)\tau) \qquad (2)$$

The m dimensional portrait constructed from the vector $v(t)$ can have the same properties as a original attractor. These properties can be made sure by Lyapunov exponent.

2. Lyapunov exponent [6][7]

The sign of Lyapunov exponent signifies chaos and the value of Lyapunov exponent measures how chaotic. A bounded dynamical system with a positive Lyapunov exponent is chaotic, and furthermore, Lyapunov exponent indicates the average rate at which predictability is lost.

As for the Lyapunov exponent, quantification does the enlargement that is a basic property, condition of the enlargement of the folding phenomenon of the chaos. The thing which took the average of the index about trajectory in time long enough is Lyapunov exponent λ, and given by Equation (3), where N denotes the number of iteration , $f(x_i)$ denotes one-dimensional map and

$$f(x_i) = x_{i+1}, \quad f'(x_i) = \frac{df(x_i)}{dx_i} \ .$$

$$\lambda = \lim_{N\to\infty} \frac{1}{\mathrm{N}} \sum_{i=0}^{\mathrm{N}-1} \log|f'(x_i)| \qquad (3)$$

If a Lyapunov exponent λ is a positive value, the distance between neighboring trajectories increases exponentially. And λ indicates the degree of sensitivity to the initial condition. On the contrary, the trajectory of the graph can be said to be stable if λ is a negative value.

3. Fractal dimension [2][8]

The geometrical structure of fractal in chaotic dynamic system has self-similarity in many case. And fractal dimension quantifies the self-similarity that is a characteristic of chaos. Fractal dimension can be estimated by box counting method and correlation integral. Correlation integral is adopted in our study because smaller memory and data are needed than those of box counting method. Fractal dimension can be calculated by following Equation(4).

$$C^m(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{\substack{i,j=1 \\ i\neq j}}^{N} I(r - |v(i) - v(j)|) \qquad (4)$$

In this equation, $I(t)$ is Heaviside function defined by Equation(5), N is the iteration number, $v(i)$ denotes the point on the attractor that exits in m dimension space and r denotes the radius of m dimensional super sphere.

$$I(t) = \begin{cases} 1 & (t \geq 0) \\ 0 & (t < 0) \end{cases} \qquad (5)$$

IV. Process of Chaotic Analysis of Economical Time Series Data [8][9]

Nikkei stock average which is calculated about stock prices of 225 brands of first section of Tokyo stock exchange and chronological order of the TOPIX (Tokyo Stock Exchange Stock Price Index) are used to investigate chaos of economical time series data.

These data are embedded into m=3 dimensional space using Equation(2) of the Takens' embedding theorem to reconstruct attractor and Figure 1 and Figure 2 respectively represent NIKKEI average attractor and TOPIX attractor. These figures show the attractor which is a characteristic of the chaos, therefore we can confirm chaos of these economical data.

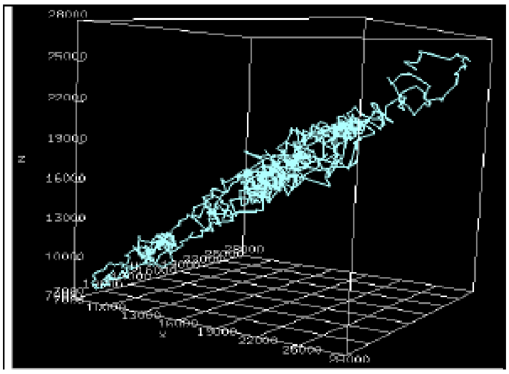

Figure 1 NIKKEI average attractor (1991~2007)
(embedded dimension: m = 3, delay time: τ =3)

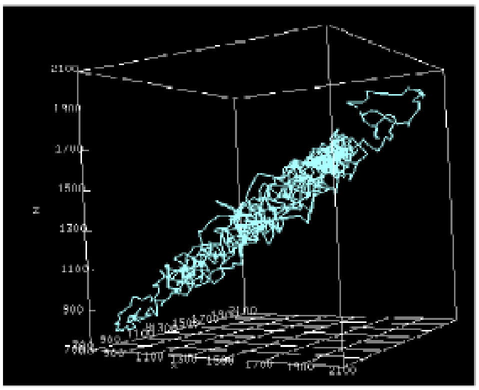

Figure 2 TOPIX attractor (1991~2007)
(m = 3, τ = 3)

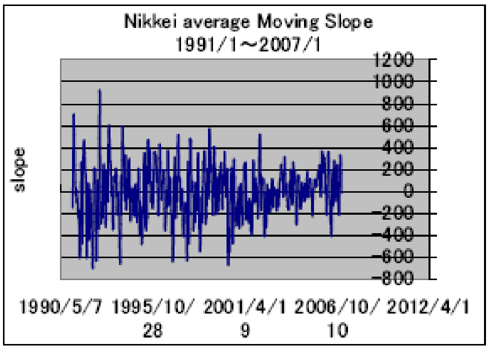

Figure 3 Nikkei moving slope

Furthermore, Lyapunov exponent is used as an index of the evaluation of chaos. The degree of leaning at each point of the graph is necessary to find a Lyapunov exponent. When there is not the expression of the graph, moving slope $x'(t)$ of item $2p$+1 is calculated by Equation(6).

$$x'(t) = \frac{-px(t-p) - \cdots - x(t-1) + x(t+1) + \cdots + px(t+p)}{p(p+1)(2p+1)/3} \quad (6)$$

The moving slope of five clauses was used this time. A result of Nikkei average is shown in Figure 3. From this result, we confirmed that the value of the moving slope changes with wide fluctuation.

Furthermore, Lyapunov exponents of Nikkei average and TOPIX can be calculated by using a moving slope and the results are shown in Table 1.

Table 1. Lyapunov exponent

	Lyapunov exponent
Nikkei average	4.784789
TOPIX	2.195722

The weekend data of Nikkei average and TOPIX from January, 1991 to January, 2007 were used for analyzing chaotic structure. As a result, it was clarified that Lyapunov exponent of Nikkei average was larger than that of TOPIX and also that Nikkei average was more complicated chaos.

V. CONCLUSION

In this study, we used finance time series data such as the Nikkei average TOPIX to confirm chaos by reconstructing attractor and calculating Lyapunov exponent and fractal dimension. We could obtain the results that Lyapunov exponents are positive number, and the fractal dimensions become the non-integer value. These results confirm chaos of the structure of the economic change. By using chaos predictions such as the local reproduction prediction law et al, we want to examine the utility of the chaos prediction.

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